

КОНТРОЛЬНА РОБОТА №3

Приклад 1. Розв'язати диференціальні рівняння

$$1.1 \quad y'(x) = \frac{3 \cos^2 2y}{\sin^2 3x}, \quad 1.2 \quad \left(4x^3 + \frac{2}{x} + 1\right)dx - (e^{2y} - 2)dy = 0,$$

$$\checkmark 1.3 \quad (3x^3 + 5x - 3)\sin^2 2y dx + x(3\sin^2 2y - 4 + 2\sin^3 2y)dy = 0,$$

$$1.4 \quad e^{2x}y' = y(2 + 4x^3e^{2x}), \quad 1.5 \quad (x + 2y - 3)y' = 1, \quad 1.6 \quad y'(x) = \frac{y}{2^x},$$

$$1.7 \quad \left(3e^{3x} - \frac{5}{x} - 8\right)dx - (2 \cdot 6^{2y} - 2\sin 2y)dy = 0,$$

$$1.8 \quad (4x^{-2} + 2x - 4)\cos^2 3y dx + x(4\cos^2 3y - 6\cos^3 3y + 9)dy = 0, \quad 1.9 \quad y'(x) = \frac{6\sin 3x}{\cos 2y},$$

$$1.10 \quad 4^{-2x}y' = y^2(6 + 4^{-2x} - 8x^34^{-2x}), \quad 1.11 \quad (-x + 3y + 1)y' = -1,$$

$$1.12 \quad (4 \cdot 2^{4x} - 2\sin 2x + 3)dx - (e^{2y} + 3y^2)dy = 0, \quad 1.13 \quad 4x^2y' = y^{-3}(6x + 3x^{-2}),$$

$$1.14 \quad \left(3e^{-2x} + 2xe^{3x} - \frac{4e^{3x}}{x}\right)\sin^2 4y dx + e^{3x}(4\sin^2 4y - 6\sin^3 4y + 4)dy = 0,$$

$$1.15 \quad (-2x + y + 2)y' = 2, \quad 1.16 \quad y'(x) = \frac{7x^{2.5}}{e^{2y}},$$

$$1.17 \quad (11x^{4.5} - 4\cos 2x + 6)dx - \left(5^{2y} + \frac{5}{\cos^2 5y}\right)dy = 0,$$

$$1.18 \quad \left(5 \cdot 4^{-2x} + 2x \cdot 4^{3x} - \frac{4^{3x}}{2x}\right)y^2 dx + 4^{3x}(4y^5 - 6y^3 + 4y^2 \cos 2y)dy = 0,$$

$$1.19 \quad -2\sin^2 2x \cdot y' = y^3(6 - 4\sin^3 2x), \quad 1.20 \quad (3x + 2y + 1)y' = -3,$$

$$1.21 \quad y'(x) = \cos^2 3y \cdot e^{3x}, \quad 1.22 \quad \left(10x^{1.5} - \frac{4}{\sin^2 4x} - 7\right)dx - \left(\frac{1}{2y} + 12\cos 6y\right)dy = 0.$$

$$1.23 \quad (4x^3 - 3x^2 - 6x^4)\sin^2 3y dx + x^3(6\sin^2 3y - 12 + 3\sin^3 3y)dy = 0,$$

$$1.24 \quad e^{2x}y' = y(2 + e^{2x} + 2x^3e^{2x}), \quad 1.25 \quad (-3x + 3y + 2)y' = 4, \quad 1.26 \quad y'(x) = \cos 4x \cdot 4^{-2x},$$

$$1.27 \quad \left(6x^{-4} + \frac{5}{x} - 11\right)dx - (3e^{-3y} + 6\sin 2y)dy = 0, \quad 1.28 \quad (4x + 5y)y' = 0.4,$$

$$1.29 \quad 5^{-3x}y' = y^{-3}(9 + 6 \cdot 5^{-3x} - 6x^55^{-3x}), \quad 1.30 \quad y'(x) = \sin 5x \cdot y^{-2}.$$

Приклад 2. Визначити частинний розв'язок диференціального рівняння

$$2.1 \quad y' + \operatorname{tg}^2 \left(-\frac{\pi}{4}x + \frac{\pi}{4}y + \pi\right) = -1, \quad \text{якщо } y(1) = 1,$$

$$2.2 \quad y'(x) = -\frac{\operatorname{tg} y \cdot \sin^2 y}{\cos^2 x \cdot \operatorname{ctg} y}, \quad \text{якщо } y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}, \quad \checkmark 2.3 \quad y'(x) = -\frac{2x(1+y^2)}{\sqrt{4-x^2}}, \quad \text{якщо } y(0) = 0,$$

$$2.26 \quad y' \cdot \operatorname{tg}^2 \left(\frac{\pi}{9}x + \frac{\pi}{3}y - \frac{\pi}{4} \right) = \frac{1}{3}, \text{ якщо } y(2,25) = 3,$$

$$2.27 \quad y'(x) = y \operatorname{ctg}(0,5x), \text{ якщо } y(\frac{\pi}{2}) = 1, \quad 2.28 \quad y'(x) = \frac{3x\sqrt{4y^2 - 1}}{2y\sqrt{9x^2 + 1}}, \text{ якщо } y(0) = 1,$$

$$2.29 \quad y'(x) = \frac{e^y}{x}, \text{ якщо } y(1) = 0, \quad 2.30 \quad y'(x) = \frac{3x(2+y^2)}{2y(2+x^2)}, \text{ якщо } y(0) = 0.$$

Приклад 3. Розв'язати диференціальні рівняння

$$3.1 \quad \frac{dy}{dx} = \frac{2xy - y^2}{x^2}, \quad 3.2 \quad y' \cos \frac{2y}{x} - \frac{y}{x} \cos \frac{2y}{x} = 1, \quad 3.3 \quad \frac{dy}{dx} = \frac{y-x}{2x}, \quad 3.4 \quad \frac{dy}{dx} = \frac{2x+3y}{-3x+2y},$$

$$3.5 \quad \frac{dy}{dx} = \frac{2xy + 3y^2}{x^2}, \quad 3.6 \quad 3y' \cos \frac{3y}{x} - \frac{3y}{x} \cos \frac{3y}{x} = 1, \quad 3.7 \quad \frac{dy}{dx} = \frac{2y^2 - x^2}{x^2 + 4xy}, \quad 3.8$$

$$\frac{dy}{dx} = \frac{x+y}{-x+3y}, \quad 3.9 \quad \frac{dy}{dx} = \frac{-2xy + 2y^2}{-x^2}, \quad 3.10 \quad -2y' \cos \frac{2y}{x} + \frac{2y}{x} \cos \frac{2y}{x} = 3, \quad 3.11$$

$$\frac{dy}{dx} = \frac{y^2 - 2x^2}{2x^2 + 2xy}, \quad 3.12 \quad \frac{dy}{dx} = \frac{4x-y}{x+2y}, \quad 3.13 \quad \frac{dy}{dx} = \frac{-2xy + 5y^2}{-x^2}, \quad 3.14$$

$$0,5y' \cos \frac{y}{2x} - \frac{y}{2x} \cos \frac{y}{2x} = 5, \quad 3.15 \quad \frac{dy}{dx} = \frac{3y^2 - 2x^2}{x^2 + 6xy}, \quad 3.16 \quad \frac{dy}{dx} = \frac{4x-3y}{3x+2y}, \quad 3.17$$

$$\frac{dy}{dx} = \frac{4xy + 3y^2}{2x^2}, \quad 3.18 \quad 2x \sin \frac{2y}{x} \cdot y' - 2y \sin \frac{2y}{x} + x = 0, \quad 3.19 \quad \frac{dy}{dx} = \frac{2y^2 - 3x^2}{-x^2 + 4xy}, \quad 3.20$$

$$\frac{dy}{dx} = \frac{-x+y}{-x+2y}, \quad 3.21 \quad \frac{dy}{dx} = \frac{4xy + 5y^2}{2x^2}, \quad 3.22 \quad 3x \sin \frac{3y}{x} \cdot y' - 3y \sin \frac{3y}{x} + 2x = 0, \quad 3.23$$

$$\frac{dy}{dx} = \frac{-y^2 - 3x^2}{2x^2 - 2xy}, \quad 3.24 \quad \frac{dy}{dx} = \frac{-4x+3y}{-3x+2y}, \quad 3.25 \quad \frac{dy}{dx} = \frac{6xy + 4y^2}{3x^2}, \quad 3.26$$

$$x \sin \frac{y}{2x} \cdot y' - y \sin \frac{y}{2x} - 6x = 0, \quad 3.27 \quad \frac{dy}{dx} = \frac{-2y^2 - x^2}{3x^2 - 4xy}, \quad 3.28 \quad \frac{dy}{dx} = \frac{6x+5y}{-5x+4y}, \quad 3.29$$

$$\frac{dy}{dx} = \frac{6xy - 2y^2}{3x^2}, \quad 3.30 \quad \frac{dy}{dx} = \frac{-y^2 + 2x^2}{3x^2 - 2xy}$$

Приклад 4. Розв'язати диференціальне рівняння

$$4.1 \quad y' + \frac{2y}{x} = 2x^2, \quad 4.2 \quad y' + 2y \cos 3x = 4 \cos 3x, \quad 4.3 \quad y' + \frac{2y}{-2x+1} = e^{2x}(-2x+1), \quad 4.4$$

$$xy' + y = xy^2, \quad 4.5 \quad y' + \frac{3y}{x} = -4x, \quad 4.6 \quad y' - 2y \cos 4x = 5 \cos 4x, \quad 4.7$$

$$y' - \frac{2y}{4x+3} = e^{3x}(4x+3)^{0.5}, \quad 4.8 \quad xy' - y = xy^2, \quad 4.9 \quad y' + \frac{4y}{x} = 2x^3, \quad 4.10$$

$$y' + 3y \cos 3x = -6 \cos 3x, \quad 4.11 \quad y' + \frac{3y}{x+2} = \frac{e^{-2x}}{(x+2)^3}, \quad 4.12 \quad y' - y = xy^2, \quad 4.13$$

$$y' + \frac{5y}{x} = -3x^2, \quad 4.14 \quad y' - 4y \cos 2x = 4 \cos 2x, \quad 4.15 \quad y' - \frac{3y}{x+4} = e^{2x}(x+4)^3, \quad 4.16$$

$$y' + \frac{y}{x} = \ln 2x \cdot y^2, \quad 4.17 \quad y' - \frac{2y}{x} = 3x^3, \quad 4.18 \quad y' + 2y \sin 2x = -4 \sin 2x, \quad 4.19$$

$$y' + \frac{4y}{2x+1} = e^{-3x}(2x+1)^{-2}, \quad 4.20 \quad y' + \frac{y}{x-1} = \frac{1}{x-1}y^2, \quad 4.21 \quad y' - \frac{3y}{x} = -2x, \quad 4.22$$

$$y' - 2y \sin 4x = 6 \sin 4x, \quad 4.23 \quad y' - \frac{4y}{2x-3} = e^{-4x}(2x-3)^2, \quad 4.24 \quad y' + \frac{y}{x} = 0.5y^2, \quad 4.25$$

$$y' - \frac{4y}{x} = 3x^2, \quad 4.26 \quad y' + 3y \sin 6x = 3 \sin 6x, \quad 4.27 \quad y' + \frac{2y}{2x+5} = e^{4x}(2x+5)^{-1}, \quad 4.28$$

$$y' + \frac{2y}{x} = x^5 y^2, \quad 4.29 \quad y' - \frac{5y}{x} = -3x^3, \quad 4.30 \quad y' - 3y \sin 3x = 9 \sin 3x.$$

Приклад 5. Розв'язати диференціальне рівняння

$$5.1 \quad (x^2 + y)dx - xdy = 0, \quad 5.2 \quad (x^2 + y^2 + 2x)dx + 2ydy = 0,$$

$$5.3 \quad (2x \cos 3y + 7x^{-2}y^2)dx + (-3x^2 \sin 3y - 14x^{-1}y)dy = 0,$$

$$5.4 \quad (-2x^{-3} \cos 3y + 10xy^{-1})dx + (-3x^{-2} \sin 3y - 5x^2y^{-2})dy = 0,$$

$$5.5 \quad (-x^{-2} \cos 3y - 4x^3y^{-2})dx + (-3x^{-1} \sin 3y + 2x^4y^{-3})dy = 0,$$

$$5.6 \quad (-x^{-2} \cos 2y - 12x^{-3}y^5)dx + (-2x^{-1} \sin 2y + 30x^{-2}y^4)dy = 0,$$

$$5.7 \quad (3x^2 \cos 4y + 15x^2y^2)dx + (-4x^3 \sin 4y + 10x^3y)dy = 0,$$

$$5.8 \quad (-3x^{-4} \cos 5y + 2x^{-3}y^3)dx + (-5x^{-3} \sin 5y - 3x^{-2}y^2)dy = 0,$$

$$5.9 \quad (4x^3 \cos 2y + 6x^{-4}y^4)dx + (-2x^4 \sin 2y - 8x^{-3}y^3)dy = 0,$$

$$5.10 \quad (-4x^{-5} \cos 3y - 2x^{-2}y^{-2})dx + (-3x^{-4} \sin 3y - 4x^{-1}y^{-3})dy = 0,$$

$$5.11 \quad (-3x^2 + 4y^2 + y)dx + (8xy + 3y^2 + x - 2y)dy = 0,$$

$$5.12 \quad (-2x^{-3} \sin 2y - x^{-2}y^2)dx + (2x^{-2} \cos 2y + 2x^{-1}y)dy = 0,$$

$$5.13 \quad (6x^2 - 4xy + y^2)dx + (-2x^2 + 2xy - 3y^2 + 2y)dy = 0,$$

$$5.14 \quad (-x^{-2} \sin 3y - y^{-2})dx + (3x^{-1} \cos 3y + 2xy^{-3})dy = 0,$$

$$5.15 \quad (-3y^2 + 4x + 2y)dx + (-6xy - 6y^2 + 2x)dy = 0,$$

$$5.16 (2x \sin 3y + 4xy^3)dx + (3x^2 \cos 3y + 6x^2y^2)dy = 0,$$

$$5.17 (-6x^2 + 4x + 3y)dx + (3x - 4y)dy = 0,$$

$$5.18 (-3x^2 \sin 2y + 6xy^{-3})dx + (-2x^3 \cos 2y - 9x^2y^{-4})dy = 0,$$

$$5.19 (6x^2 - 2xy + 7y^2)dx + (-x^2 + 14xy + 4y)dy = 0,$$

$$5.20 (-3x^{-4} \sin 4y + 2x^{-3}y^3)dx + (4x^{-3} \cos 4y - 3x^{-2}y^2)dy = 0,$$

$$5.21 (6xy - 5y^2 - 3y)dx + (3x^2 - 10xy + 3y^2 - 3x - 6y)dy = 0,$$

$$5.22 (4x^3 \sin 2y + 8x^3y^{-2})dx + (2x^4 \cos 2y - 4x^4y^{-3})dy = 0,$$

$$5.23 (9x^2 - 7y^2 + 3y)dx + (-14xy - 9y^2 + 3x)dy = 0,$$

$$5.24 (-4x^{-5} \sin 3y + 10x^{-3}y^4)dx + (3x^{-4} \cos 3y - 20x^{-2}y^3)dy = 0,$$

$$5.25 (-9x^2 + 2xy + 7y)dx + (x^2 + 7x - 2y)dy = 0,$$

$$5.26 (-2x^{-3} \sin 4y + 12x^3y^{-2})dx + (4x^{-2} \cos 4y - 6x^4y^{-3})dy = 0,$$

$$5.27 (x^2 + x + 2y)dx + xdy = 0, 5.28 xydx + (x^2 + 1)dy = 0,$$

$$5.29 (-2x^2 + 2x + 2y)dx - xdy = 0, 5.30 -xydx + (2x^2 + 3)dy = 0.$$

Приклад 6. Визначити розв'язок задачі Коші, якщо

$$6.1 xy'' - y' = 9x^2e^{3x}, y(0) = \frac{1}{3} \text{ i } y'(1) = e^3,$$

$$6.2 y'' + \operatorname{tg} x \cdot y' = 2 \cos x, y(0) = 0 \text{ i } y'(0) = -1,$$

$$\checkmark 6.3 xy'' - y' = x^2e^{0.5x}, y(0) = -6 \text{ i } y'(1) = e^{0.5},$$

$$6.4 y'' + \operatorname{tg} x \cdot y' = -4 \cos x, y(0) = -1 \text{ i } y'(0) = 2,$$

$$6.5 xy'' - y' = -2x^2e^x, y(0) = 1 \text{ i } y'(1) = e,$$

$$6.6 y'' + \operatorname{tg} x \cdot y' = 5 \cos x, y(0) = 6 \text{ i } y'(0) = 2,$$

$$6.7 xy'' - y' = -8x^2e^{-2x}, y(0) = 1 \text{ i } y'(1) = 3e^{-2},$$

$$6.8 y'' + \operatorname{tg} x \cdot y' = -3 \cos x, y(0) = 1 \text{ i } y'(0) = -2,$$

$$6.9 y' = 2(x+1) + 2\pi \sin\left(\frac{\pi x}{4}\right) - 2\pi \cos\left(\frac{\pi x}{6}\right) \text{ i } y(0) = -5,$$

$$6.10 y' = 2(x+1)^{-1} + 6\pi \sin(2\pi x) - 12\pi \cos(3\pi x) \text{ i } y(0) = -2,$$

$$6.11 y' = 3(x-1)^2 - 16\pi \sin(4\pi x) + 2\pi \cos(2\pi x) \text{ i } y(0) = 2,$$

$$6.12 y' = -(x+2)^{-2} - 27\pi \sin(3\pi x) + 12\pi \cos(4\pi x) \text{ i } y(0) = 5.5,$$

$$6.13 y' = -(x+4)^{-0.5} + 25\pi \sin(5\pi x) - 12\pi \cos(3\pi x) \text{ i } y(0) = -5,$$

$$6.14 y' = 1.5(x+1)^{0.5} + 2\pi \sin\left(\frac{\pi x}{3}\right) - 2\pi \cos\left(\frac{\pi x}{4}\right) \text{ i } y(0) = -2,$$

$$6.15 y' = 0.5(x+9)^{-0.5} - 2\pi \sin\left(\frac{\pi x}{5}\right) + \pi \cos\left(\frac{\pi x}{6}\right) \text{ i } y(0) = 8,$$

- 5.16** $(2x \sin 3y + 4xy^3)dx + (3x^2 \cos 3y + 6x^2y^2)dy = 0,$
- 5.17** $(-6x^2 + 4x + 3y)dx + (3x - 4y)dy = 0,$
- 5.18** $(-3x^2 \sin 2y + 6xy^{-3})dx + (-2x^3 \cos 2y - 9x^2y^{-4})dy = 0,$
- 5.19** $(6x^2 - 2xy + 7y^2)dx + (-x^2 + 14xy + 4y)dy = 0,$
- 5.20** $(-3x^{-4} \sin 4y + 2x^{-3}y^3)dx + (4x^{-3} \cos 4y - 3x^{-2}y^2)dy = 0,$
- 5.21** $(6xy - 5y^2 - 3y)dx + (3x^2 - 10xy + 3y^2 - 3x - 6y)dy = 0,$
- 5.22** $(4x^3 \sin 2y + 8x^3y^{-2})dx + (2x^4 \cos 2y - 4x^4y^{-3})dy = 0,$
- 5.23** $(9x^2 - 7y^2 + 3y)dx + (-14xy - 9y^2 + 3x)dy = 0,$
- 5.24** $(-4x^{-5} \sin 3y + 10x^{-3}y^4)dx + (3x^{-4} \cos 3y - 20x^{-2}y^3)dy = 0,$
- 5.25** $(-9x^2 + 2xy + 7y)dx + (x^2 + 7x - 2y)dy = 0,$
- 5.26** $(-2x^{-3} \sin 4y + 12x^3y^{-2})dx + (4x^{-2} \cos 4y - 6x^4y^{-3})dy = 0,$
- 5.27** $(x^2 + x + 2y)dx + xdy = 0, \quad \text{5.28 } xydx + (x^2 + 1)dy = 0,$
- 5.29** $(-2x^2 + 2x + 2y)dx - xdy = 0, \quad \text{5.30 } -xydx + (2x^2 + 3)dy = 0.$

Приклад 6. Визначити розв'язок задачі Коші, якщо

6.1 $xy'' - y' = 9x^2e^{3x}, \quad y(0) = \frac{1}{3} \text{ i } y'(1) = e^3,$

6.2 $y'' + \operatorname{tg} x \cdot y' = 2 \cos x, \quad y(0) = 0 \text{ i } y'(0) = -1,$

6.3 $xy'' - y' = x^2e^{0.5x}, \quad y(0) = -6 \text{ i } y'(1) = e^{0.5},$

6.4 $y'' + \operatorname{tg} x \cdot y' = -4 \cos x, \quad y(0) = -1 \text{ i } y'(0) = 2,$

6.5 $xy'' - y' = -2x^2e^x, \quad y(0) = 1 \text{ i } y'(1) = e,$

6.6 $y'' + \operatorname{tg} x \cdot y' = 5 \cos x, \quad y(0) = 6 \text{ i } y'(0) = 2,$

6.7 $xy'' - y' = -8x^2e^{-2x}, \quad y(0) = 1 \text{ i } y'(1) = 3e^{-2},$

6.8 $y'' + \operatorname{tg} x \cdot y' = -3 \cos x, \quad y(0) = 1 \text{ i } y'(0) = -2,$

6.9 $y' = 2(x+1) + 2\pi \sin\left(\frac{\pi x}{4}\right) - 2\pi \cos\left(\frac{\pi x}{6}\right) \text{ i } y(0) = -5,$

6.10 $y' = 2(x+1)^{-1} + 6\pi \sin(2\pi x) - 12\pi \cos(3\pi x) \text{ i } y(0) = -2,$

6.11 $y' = 3(x-1)^2 - 16\pi \sin(4\pi x) + 2\pi \cos(2\pi x) \text{ i } y(0) = 2,$

6.12 $y' = -(x+2)^{-2} - 27\pi \sin(3\pi x) + 12\pi \cos(4\pi x) \text{ i } y(0) = 5,5,$

6.13 $y' = -(x+4)^{-0.5} + 25\pi \sin(5\pi x) - 12\pi \cos(3\pi x) \text{ i } y(0) = -5,$

6.14 $y' = 1,5(x+1)^{0.5} + 2\pi \sin\left(\frac{\pi x}{3}\right) - 2\pi \cos\left(\frac{\pi x}{4}\right) \text{ i } y(0) = -2,$

6.15 $y' = 0,5(x+9)^{-0.5} - 2\pi \sin\left(\frac{\pi x}{5}\right) + \pi \cos\left(\frac{\pi x}{6}\right) \text{ i } y(0) = 8,$

$$6.16 \quad y' = \frac{4}{3}(x+8)^{-\frac{1}{3}} - 0,2\pi \sin\left(\frac{\pi x}{6}\right) + 0,125\pi \cos\left(\frac{\pi x}{8}\right) \text{ i } y(0) = 8,$$

Розв'язати диференціальне рівняння

$$\begin{aligned} 6.17 \quad xy''' - y'' &= -x^2, \quad 6.18 \quad yy'' + y'^2 = 2, \quad 6.19 \quad x^2y''' - xy'' = 5x^4, \quad 6.20 \quad yy'' - y'^2 = 1, \quad 6.21 \\ xy''' - y'' &= 6x^{1,5}, \quad 6.22 \quad yy'' - 0,5y'^2 = 1, \quad 6.23 \quad x^2y''' - xy'' = -8x^{2,5}, \quad 6.24 \quad yy'' + y'^2 = 0,5, \\ 6.25 \quad x^4y''' - x^3y'' &= -6x, \quad 6.26 \quad yy'' - y'^2 = -2, \quad 6.27 \quad x^3y''' - x^2y'' = 5x^{0,5}, \quad 6.28 \\ yy'' - 0,5y'^2 &= -2, \quad 6.29 \quad y''' - x^{-1}y'' = -2x^{-0,5}, \quad 6.30 \quad yy'' + y'^2 = 4. \end{aligned}$$

Приклад 7. Визначити загальний розв'язок лінійного однорідного диференціального рівняння

$$\begin{aligned} 7.1 \quad \text{a)} \quad y'' + y' - 2y &= 0 \quad \text{б)} \quad y''' + 3y'' - 10y' - 24y = 0, \quad 7.2 \quad \text{а)} \quad y'' + 6y' + 13y = 0, \quad \text{б)} \\ y''' + 6y'' + 21y' + 26y &= 0, \quad 7.3 \quad \text{а)} \quad y'' - y' - 2y = 0, \quad \text{б)} \quad y''' - 5y'' + 2y' + 8y = 0, \quad 7.4 \quad \text{а)} \\ y'' + 4y' + 13y &= 0, \quad \text{б)} \quad y''' - 3y'' + 16y' + 20y = 0, \quad 7.5 \quad \text{а)} \quad y''' - 3y'' + 16y' + 20y = 0, \quad \text{б)} \\ y''' - 2y'' - 13y' - 10y &= 0, \quad 7.6 \quad \text{а)} \quad y'' + 2y' + 17y = 0, \quad \text{б)} \quad y'' + 2y'' + 26y' = 0, \quad 7.7 \quad \text{а)} \\ y'' + y' - 6y &= 0, \quad \text{б)} \quad y''' - 13y' - 12y = 0, \quad 7.8 \quad \text{а)} \quad y'' - 2y' + 5y = 0, \quad \text{б)} \\ y''' - 7y'' + 31y' - 25y &= 0, \quad 7.9 \quad \text{а)} \quad y'' - 6y' + 8y = 0, \quad \text{б)} \quad y'' + 5y'' - 2y' - 24y = 0, \quad 7.10 \quad \text{а)} \\ y'' - 4y' + 13y &= 0, \quad \text{б)} \quad y'' + 6y'' + 9y' - 50y = 0, \quad 7.11 \quad \text{а)} \quad y'' - 2y' - 8y = 0, \quad \text{б)} \\ y''' - 7y'' + 7y' + 15y &= 0, \quad 7.12 \quad \text{а)} \quad y'' + 4y' + 20y = 0, \quad \text{б)} \quad y'' - 18y'' - 11y' + 58y = 0, \\ 7.13 \quad \text{а)} \quad y'' + y' - 6y &= 0, \quad \text{б)} \quad y''' - 2y'' - 5y' + 6y = 0, \quad 7.14 \quad \text{а)} \quad y'' - 6y' + 34y = 0, \quad \text{б)} \\ y''' + 7y'' + 40y' + 34y &= 0, \quad 7.15 \quad \text{а)} \quad y'' - 6y' + 5y = 0, \quad \text{б)} \quad y'' + 4y'' + y' - 6y = 0, \quad 7.16 \quad \text{а)} \\ y'' - 4y' + 40y &= 0, \quad \text{б)} \quad y''' - 9y'' + 40y' - 32y = 0, \quad 7.17 \quad \text{а)} \quad y'' + y' - 30y = 0, \quad \text{б)} \\ y''' - 5y'' + 12y' - 8y &= 0, \quad 7.18 \quad \text{а)} \quad y'' + 12y' + 100y = 0, \quad \text{б)} \quad y'' - 4y'' - 4y' + 16y = 0, \\ 7.19 \quad \text{а)} \quad y'' - 22y' + 121y &= 0, \quad \text{б)} \quad y''' + 4y'' + 14y' + 20y = 0, \quad 7.20 \quad \text{а)} \quad y'' - 4y' - 32y = 0, \\ \text{б)} \quad y''' - 5y'' + 4y' + 10y &= 0, \quad 7.21 \quad \text{а)} \quad y'' + 10y' + 61y = 0, \quad \text{б)} \quad y'' - y'' - 9y' + 9y = 0, \quad 7.22 \\ \text{а)} \quad y'' - 26y' + 169y &= 0, \quad \text{б)} \quad y''' - 6y'' + 5y' + 12y = 0, \quad 7.23 \quad \text{а)} \quad y'' - 4y' - 12y = 0, \quad \text{б)} \\ y''' - 6y'' + 13y' - 10y &= 0, \quad 7.24 \quad \text{а)} \quad y'' - 12y' + 52y = 0, \quad \text{б)} \quad y'' + 3y'' - 4y' - 12y = 0, \\ 7.25 \quad \text{а)} \quad y'' + 2y' - 24y &= 0, \quad \text{б)} \quad y''' + 8y'' + 11y' - 20y = 0, \quad 7.26 \quad \text{а)} \quad y'' + 10y' + 74y = 0, \quad \text{б)} \\ y''' + 11y'' + 84y' + 74y &= 0, \quad 7.27 \quad \text{а)} \quad y'' - 16y' + 64y = 0, \quad \text{б)} \quad y'' - 13y'' + 12y'' = 0, \quad 7.28 \\ \text{а)} \quad y'' + 6y' + 9y &= 0, \quad \text{б)} \quad y''' + 14y'' + 49y' = 0, \quad 7.29 \quad \text{а)} \quad y'' - 8y' + 16y = 0, \quad \text{б)} \\ y''' - 3y'' + 12y' - 10y &= 0, \quad 7.30 \quad \text{а)} \quad y'' + 10y' + 25y = 0, \quad \text{б)} \quad y'' + 2y'' + 21y' - 58y = 0. \end{aligned}$$

Приклад 8. Визначити загальний розв'язок неоднорідного диференціального рівняння зі сталими коефіцієнтами

$$\begin{aligned} 8.1 \quad y'' - y' - 2y &= 3x^2 - 6, \quad 8.2 \quad y'' + 3y' - 4y = 12xe^{-x}, \quad 8.3 \quad y'' + 2y' - 3y = 4x^2 - 8x, \\ 8.4 \quad y'' + y' - 6y &= 8xe^x, \quad 8.5 \quad y'' + y' - 12y = 14x^2 - 28, \quad 8.6 \quad y'' - 4y' + 3y = -5xe^{2x}, \\ 8.7 \quad y'' - 2y' - 8y &= 6x^2 - 12x, \quad 8.8 \quad y'' - 5y' + 4y = 9xe^{-2x}, \quad 8.9 \quad y'' - 2y' - 3y = -4x^2 - 8, \\ 8.10 \quad y'' - 4y' - 12y &= 7,5xe^{3x}, \quad 8.11 \quad y'' - 4y' + 3y = 6x^2 - 4x, \end{aligned}$$

- 8.12** $y'' - 2y' - 15y = 14xe^{-2x}$, **8.13** $y'' - 4y = -8x^2 + 16$, **8.14** $y'' - 6y' + 5y = 12xe^{2x}$,
8.15 $y'' - 3y' + 2y = -3x^2 - 5x$, **8.16** $y'' - 2y' - 15y = 24xe^{3x}$,
8.17 $y'' - 5y' + 6y = 6x^2 + 7x + 19$, **8.18** $y'' - 4y' + 3y = (6x + 2)e^{4x}$,
8.19 $y'' - 2y' - 8y = -5xe^{-x}$, **8.20** $y'' - 4y' - 5y = -16xe^x$, **8.21** $y'' - 7y' + 6y = 7xe^{-x}$,
8.22 $y'' - 9y' + 14y = 18xe^x$, **8.23** $y'' - 6y' + 8y = 15xe^{-x}$, **8.24** $y'' - 5y' + 6y = 6xe^x$,
8.25 $y'' - y' - 6y = 18xe^x$, **8.26** $y'' - 3y' - 10y = -12xe^{2x}$,
8.27 $y'' - 8y' + 16y = (3x + 2)e^{4x}$, **8.28** $y'' - 8y' + 15y = (3x^2 + 1)e^{-2x}$,
8.29 $y'' - 8y' = -48x^2 - 4x + 14$, **8.30** $y'' - 5y' + 6y = 5\sin 4x$.

Приклад 9. Розв'язати систему диференціальних рівнянь

$9.1 \begin{cases} \frac{dy_1}{dx} = y_1 - y_2, \\ \frac{dy_2}{dx} = -4y_1 + y_2. \end{cases}$	$9.2 \begin{cases} \frac{dy_1}{dx} = y_1 + y_2, \\ \frac{dy_2}{dx} = -4y_1 + y_2 \end{cases}$	$9.3 \begin{cases} \frac{dy_1}{dx} = 3y_1 + 2y_2, \\ \frac{dy_2}{dx} = 3y_1 - 2y_2. \end{cases}$
$9.4 \begin{cases} \frac{dy_1}{dx} = 2y_1 + 3y_2, \\ \frac{dy_2}{dx} = -3y_1 + 2y_2. \end{cases}$	$9.5 \begin{cases} \frac{dy_1}{dx} = 4y_1 - 4y_2, \\ \frac{dy_2}{dx} = -2y_1 + 2y_2. \end{cases}$	$9.6 \begin{cases} \frac{dy_1}{dx} = 4y_1 - y_2, \\ \frac{dy_2}{dx} = 17y_1 + 2y_2. \end{cases}$
$9.7 \begin{cases} \frac{dy_1}{dx} = -y_1 + 4y_2, \\ \frac{dy_2}{dx} = y_1 + 2y_2. \end{cases}$	$9.8 \begin{cases} \frac{dy_1}{dx} = 5y_1 + y_2, \\ \frac{dy_2}{dx} = -5y_1 + 3y_2. \end{cases}$	$9.9 \begin{cases} \frac{dy_1}{dx} = -2y_1 + 3y_2, \\ \frac{dy_2}{dx} = 2y_1 - y_2. \end{cases}$
$9.10 \begin{cases} \frac{dy_1}{dx} = y_1 + 4y_2, \\ \frac{dy_2}{dx} = -2y_1 - 3y_2. \end{cases}$	$9.11 \begin{cases} \frac{dy_1}{dx} = -3y_1 + 2y_2, \\ \frac{dy_2}{dx} = 3y_1 + 2y_2. \end{cases}$	$9.12 \begin{cases} \frac{dy_1}{dx} = -2y_1 + 8y_2, \\ \frac{dy_2}{dx} = -2y_1 - 2y_2. \end{cases}$
$9.13 \begin{cases} \frac{dy_1}{dx} = -4y_1 + 3y_2, \\ \frac{dy_2}{dx} = 2y_1 + y_2. \end{cases}$	$9.14 \begin{cases} \frac{dy_1}{dx} = -3y_1 + 13y_2, \\ \frac{dy_2}{dx} = -2y_1 - y_2. \end{cases}$	$9.15 \begin{cases} \frac{dy_1}{dx} = -5y_1 + 7y_2, \\ \frac{dy_2}{dx} = y_1 + y_2. \end{cases}$
$9.16 \begin{cases} \frac{dy_1}{dx} = -4y_1 + 9y_2, \\ \frac{dy_2}{dx} = -2y_1 + 2y_2. \end{cases}$	$9.17 \begin{cases} \frac{dy_1}{dx} = 5y_1 - 3y_2, \\ \frac{dy_2}{dx} = -4y_1 + 6y_2. \end{cases}$	$9.18 \begin{cases} \frac{dy_1}{dx} = 2y_1 - 2y_2, \\ \frac{dy_2}{dx} = -2y_1 - y_2. \end{cases}$